Global Illumination Basics

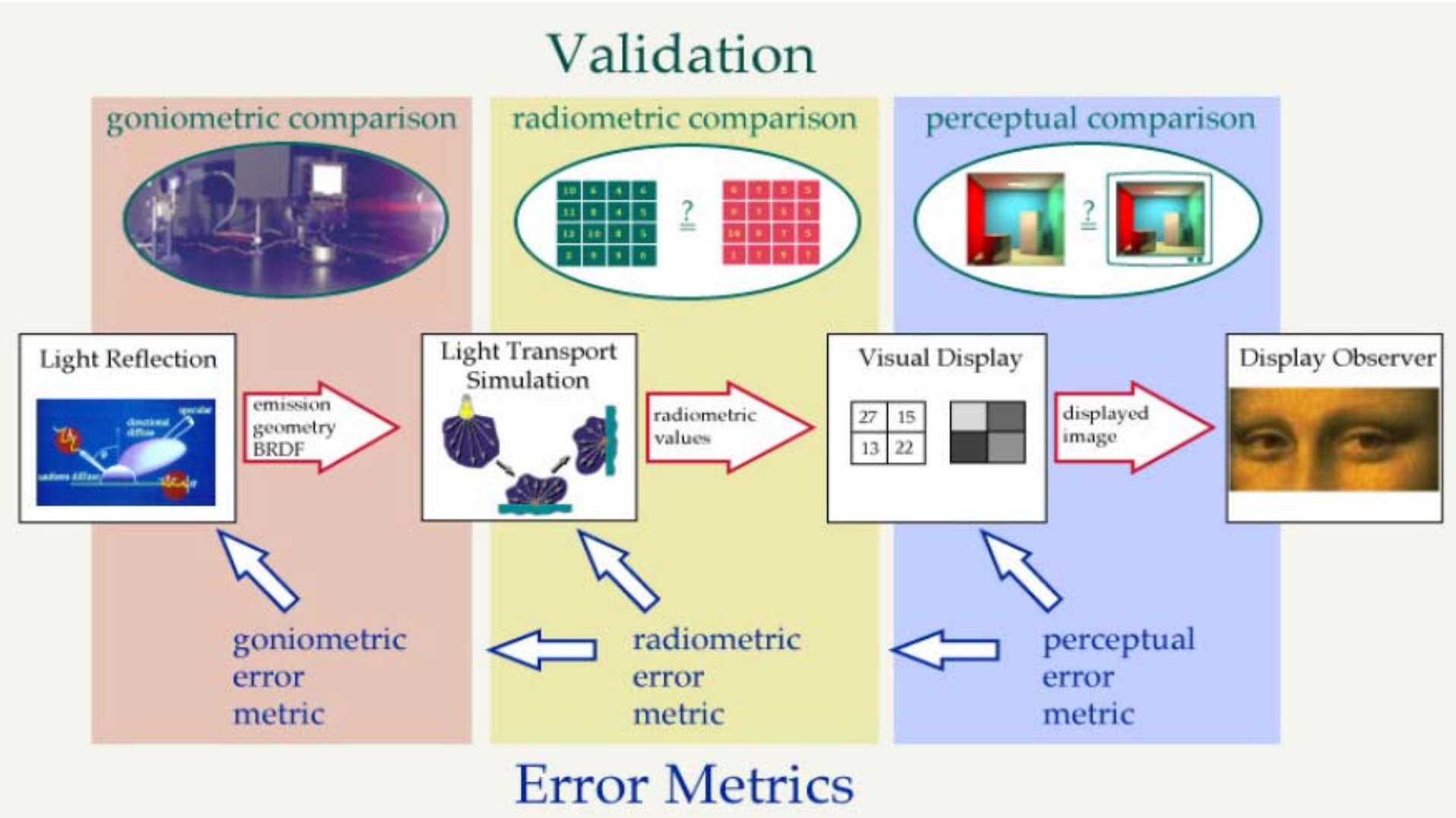
Outline

- A framework of physically based rendering
- Radiometry and photometry
- BRDF and local reflection integral

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Physically Based Rendering



Physically Based Rendering

- **Acquisition and modeling of local light reflection model**
  - **Measurement and representation**
  - **Output: Physical quantities**
- **Light transport simulation**
  - **Solving the rendering equation**
    - **Ray tracing, radiosity…**
  - **Output: Physical quantities**
- **Image display**
  - **Mapping simulated scene radiances to display radiances to produce a visual image**
  - **Output: Perceptual results**

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Radiometry

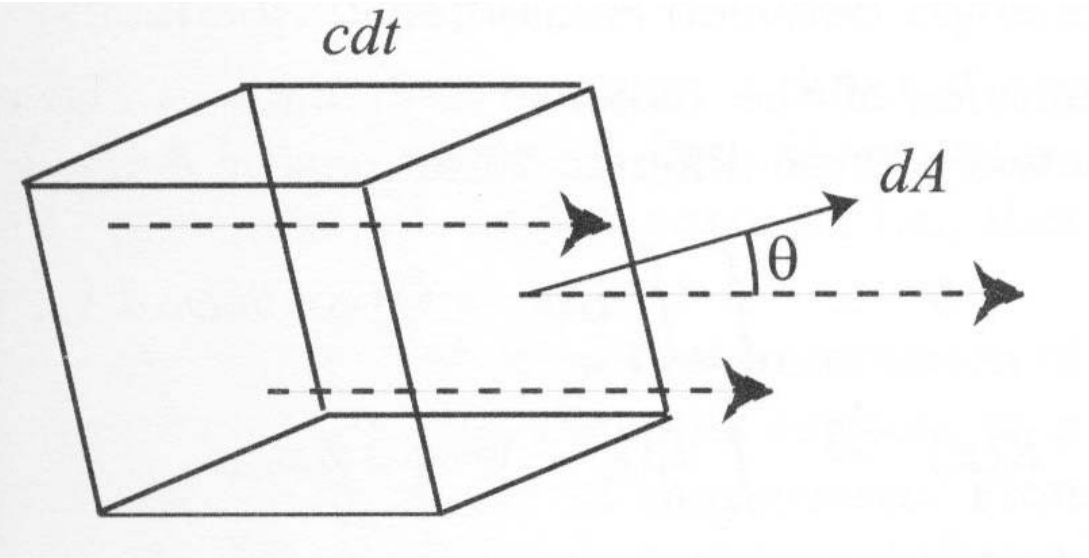
- **Radiometry is the science about radiant energy transfers**
- **Two concepts:**
  - **Flux, for energy transport**
  - **Solid angle, for defining a direction in 3D**

Radiant power (Flux)

- **Radiant power (Flux) is the fundamental radiometric quantity**
- **Denoted as Φ**
- **Expressed in *Watt* (*W*) (*Joule/sec*)**
- **How much total energy flows from/to/through a surface per unit time.**

- ***Ex. The light source emits 50W radiant power***
- ***Ex. 20W of radiant power is incident on a table.***

Particle Model and Flux 1/2



- **The number of particles flowing across the surface is**

$$p(x)cdtdA\cos\theta$$

- ***p(x)*: density of light paricles**
- **c: speed of light**

Particle Model and Flux 2/2

- **Consider all possible directions and all possible wavelengths, the number of particles**

$$N = p(x,\omega,\lambda)cdtdA\cos\theta d\omega d\lambda$$

- **Flux: the energy of the particles per unit time.**

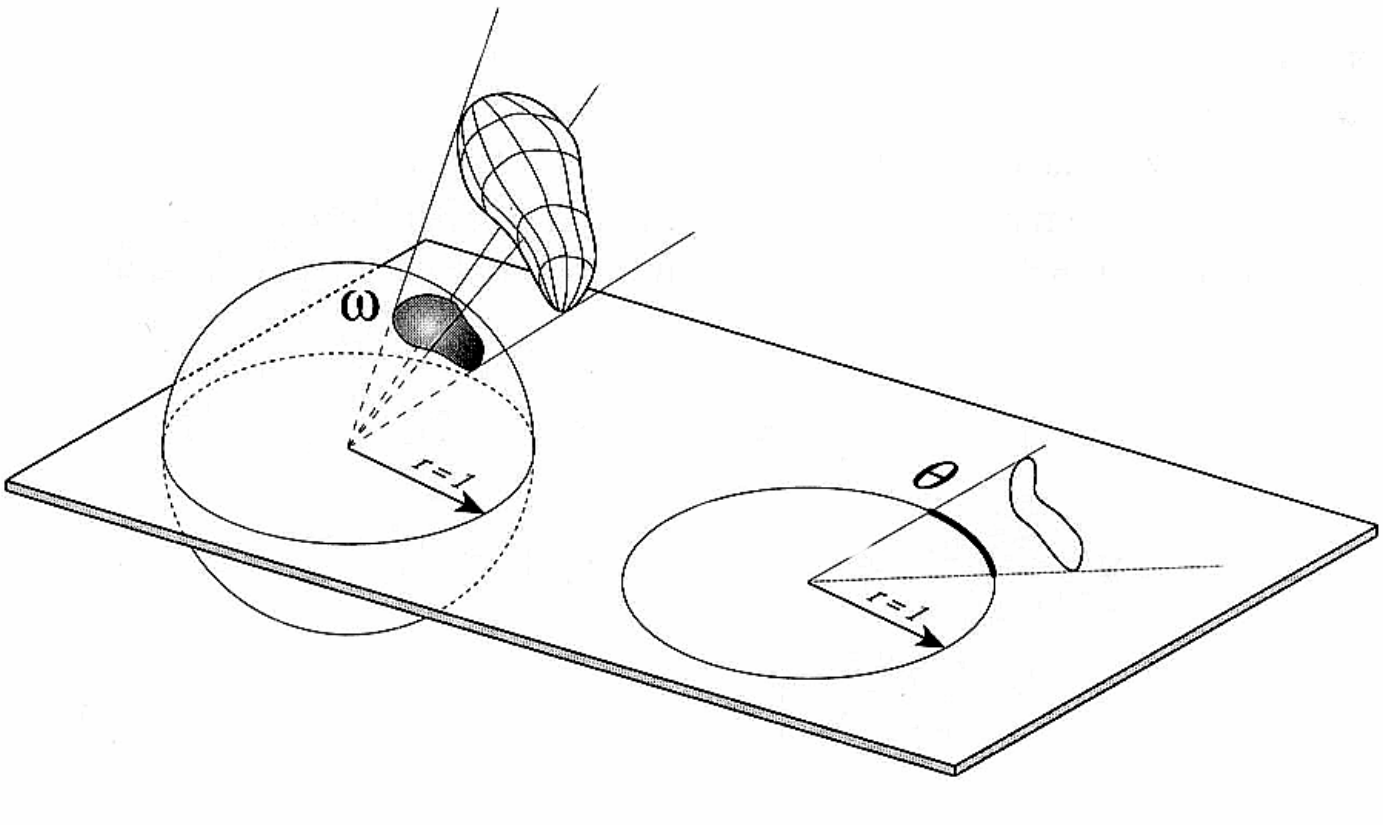
$$\Phi \propto p(x,\omega,\lambda)dA\cos\theta d\omega d\lambda$$

$$\frac{\Phi}{dA\cos\theta d\omega} \propto p(x,\omega,\lambda)d\lambda$$

Solid Angle

- **A solid angle is…**
  - **a 3-dimensional extension of the angle between two lines**
  - **a measure the portion of space occupied by an object as seen from a point**
  - **Fact: Sun and Moon occupy earth's sky hemisphere area for about 0.002 %**

Solid Angle VS. Angle

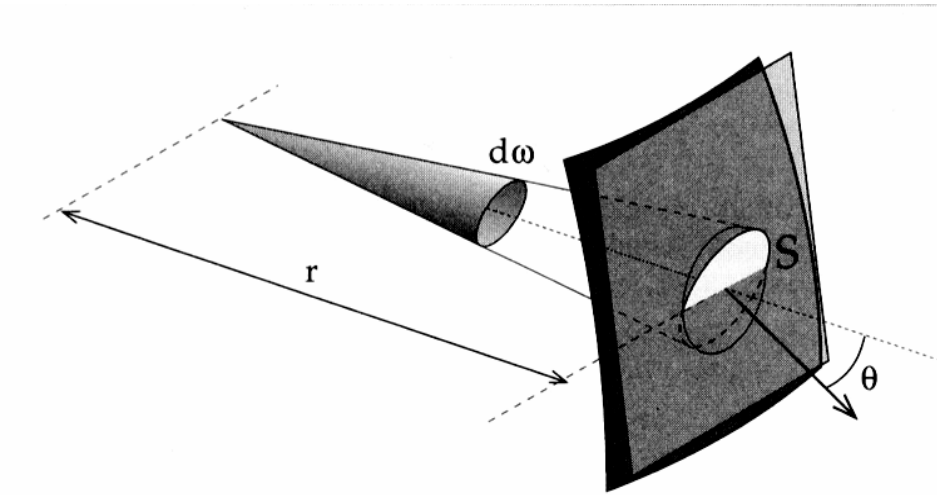


Solid Angle VS. Angle

- **"*Angle* subtended by an object is the *arc length* of the central projection of the object onto a unit circle"**
  - **Unit: radians, degrees**
  - **A full circle occupies 2π angle**
- **"The *solid angle* subtended by an object from a point P is the *area of the projection* of the object onto the unit sphere centered at P"**
  - **Unit: steradians or degrees^2**
  - **Thus the sun occupies $2/100000\pi$ solid angle.**
  - **A hemisphere's subtended solid angle is 2π, and the entire sphere is 4π**

A Small Area's Solid Angle

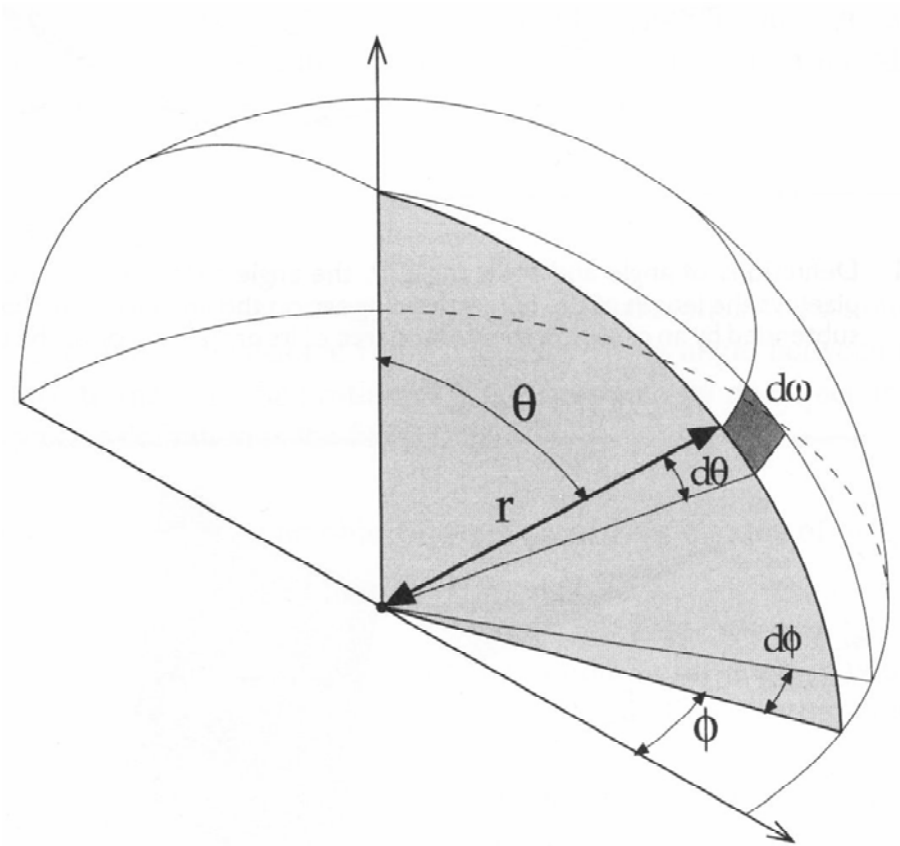
- **A small patch S with a small area dA**



$$d\omega = \frac{dA\cos\theta}{r^2}$$

Solid Angle and Spherical Coordinates

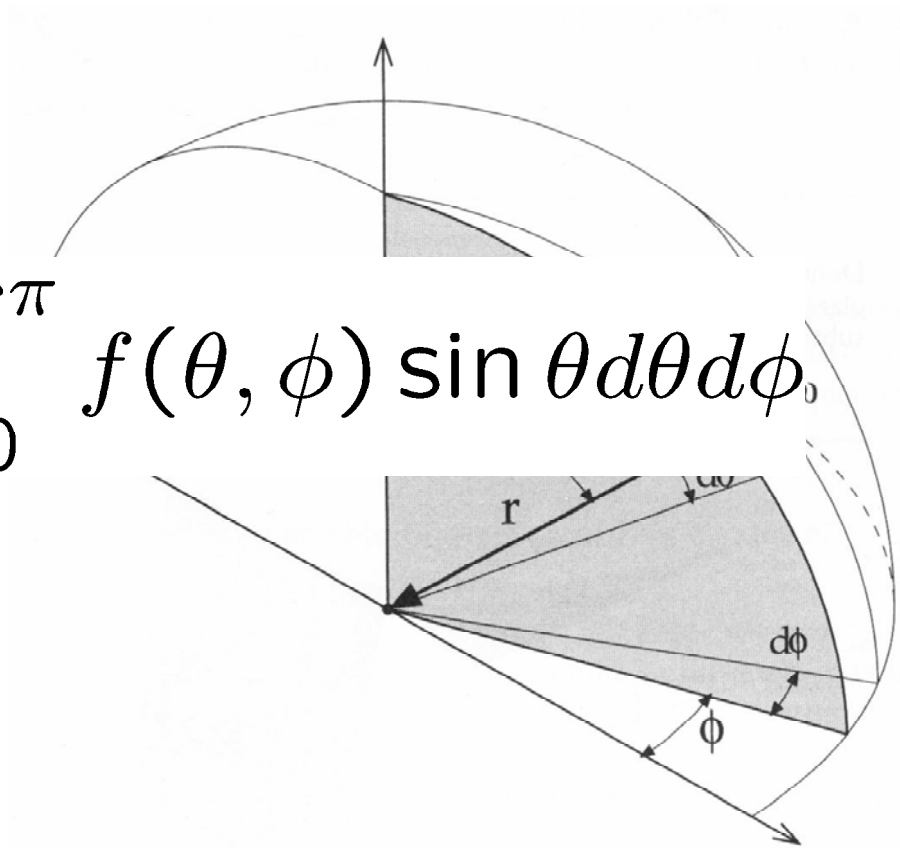
- **A useful domain for solid angle**
- **$d\omega = \sin\theta \, d\theta \, d\phi$**



Solid Angle and Spherical Coordinates

- **A useful domain for solid angle**
- **$d\omega = \sin\theta\, d\theta\, d\phi$**

$$\int_{\Gamma} f(\vec{\omega})\, d\vec{\omega} = \int_0^{2\pi} \int_0^{\pi} f(\theta, \phi) \sin\theta\, d\theta\, d\phi$$



Solid Angle and Radiance

- **The relevant quantity used to describe radiant energy transfer is *radiance***
  - **Usually denoted by *L***
  - **The amount of energy traveling**
  - **at some point**
  - **in a specified direction**
  - **per unit area perpendicular to the direction of travel**
  - **per unit solid angle**
  - **per unit time**

  - **Let's see a more intuitive figure**

$$d^3Q = L(x, \theta, \phi) dA_x cos\theta d\vec{\omega} dt$$

Solid Angle and Radiance

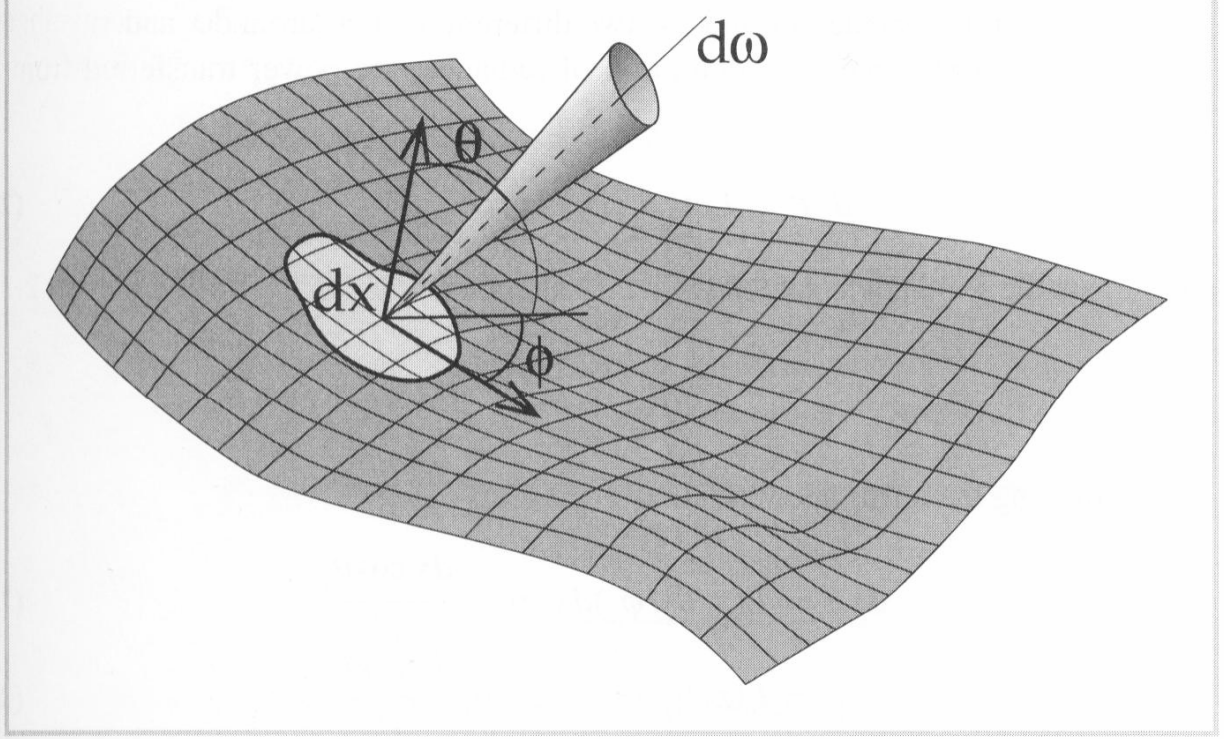


FIGURE 2.4 Definition of radiance.

Important Properties of Radiance

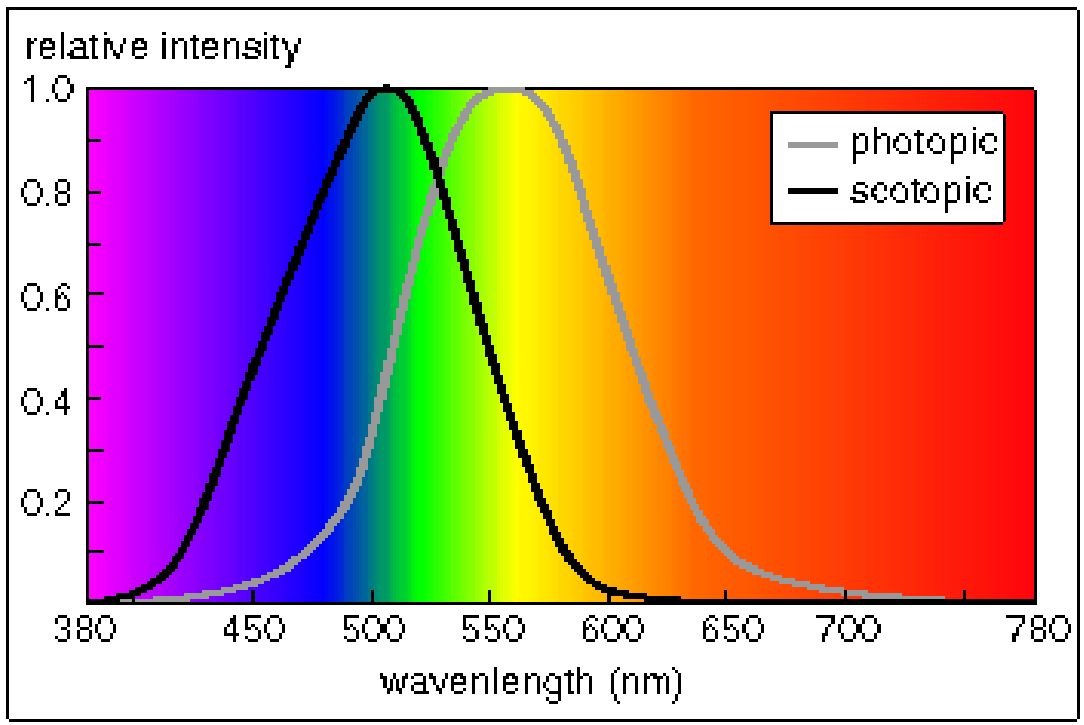
- **I: For two mutually visible points x and y, the radiance leaving point x in direction of point y is the same as the radiance impinging on point y from the direction of point x**
  - **Meaning:** radiance along a straight line doesn't change
  - **and we can "trace rays of light" in reversed direction**
- **II: Sensors such as eyes and cameras are sensitive to radiance**

Photometry

- **Considering HVS**
  - **Radiometry concepts may be adjusted to represent the perceptions of a statistically standard human observer to objective radiometric quantities**
- **Recall the luminous efficiency curve**
- **Previously mentioned radiometry quantities can map to photometry quantities by integrate spectral radiance against the luminous efficiency curve**
  - **Because the curve is usually normalized, we have to multiply the result by a constant, 680 lumen / watt**

The Lambda curve

- **The luminous efficiency curve of statistically average human eyes**
- **The curve changes as lighting condition changes**



The Lambda curve *V(λ)*

$$Q_v = K_m \int_{380}^{780} V(\lambda) Q_e(\lambda) d\lambda$$

Radiometry		Photometry	
Joule(J)	Energy	Luminous energy	Tabolt
Watt(W)	Radiant power	Luminous power	Tabolt/s Lumens(lm)
W/m^2	Radiosity/ Irradiance	Luminosity/ Illuminance	Lux(lm/m^2)
W/sr	Intensity	Luminance Intensity	Lm/sr candle(cd) candela
$W/m^2/sr$	Radiance	Luminance	cd/m^2 $lm/m^2/sr$ Nit

Outline

- A framework of physically based rendering
- Radiometry and photometry
- BRDF and local reflection integral
  - Reflection model
  - BRDF

Reflection Model

- **Definition:**

  Reflection is the process by which light incident on a surface interacts with the surface such that it leaves

  on the incident side without change in frequency.
- **There are other interactions between light and matter**
  - **Absorption**
  - **Transmission**
  - **Scattering**

BRDF

- **Bidirectional Reflectance Distribution Function**
  - **A function that models the reflection behavior**
- **BTDF for transmission**
- **BSDF = BRDF + BTDF**

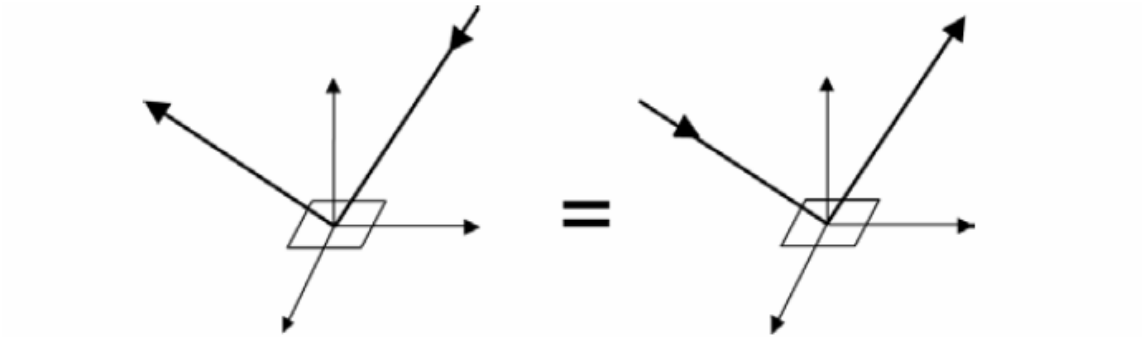
BRDF Definition

$$f_\lambda(x, \omega_{in}, \omega_{out}) = \frac{dL(x, \omega_{out})}{dE(x, \omega_{in})}$$

- **BRDF, f is the ratio of**
  - **the outgoing differential radiance and**
  - **the incoming differential irradiance**
- **Unit is st^{-1}**
- **BRDF is always greater or equal to 0**
- **Wavelength-dependent**
- **A 4D function for each point**
- **A linear function w.r.t. all incident directions**

Properties of BRDF

- **2 important properties must hold**
  - **Reciprocity rule (symmetry)**



$$f_\lambda(x, \omega_{in}, \omega_{out}) = f_\lambda(x, \omega_{out}, \omega_{in})$$

  - **Energy conservation**
    - **The total amount of energy reflected must be less than or equal to the energy received**

BRDF

- **Calculate outgoing radiance from BRDF**

$$
\begin{aligned}
f_\lambda(x, \omega_{in}, \omega_{out}) &= \frac{dL(x, \omega_{out})}{dE(x, \omega_{in})} \\
dL(x, \omega_{out}) &= f_\lambda(x, \omega_{in}, \omega_{out}) dE(x, \omega_{in}) \\
L(x, \omega_{out}) &= \int_{\Omega_x} f_\lambda(x, \omega_{in}, \omega_{out}) dE(x, \omega_{in}) \\
L(x, \omega_{out}) &= \int_{\Omega_x} f_\lambda(x, \omega_{in}, \omega_{out}) L(x, \omega_{in}) cos(N_x, \omega_{in}) d\omega
\end{aligned}
$$

Local reflection integral

- **BRDF in practice**

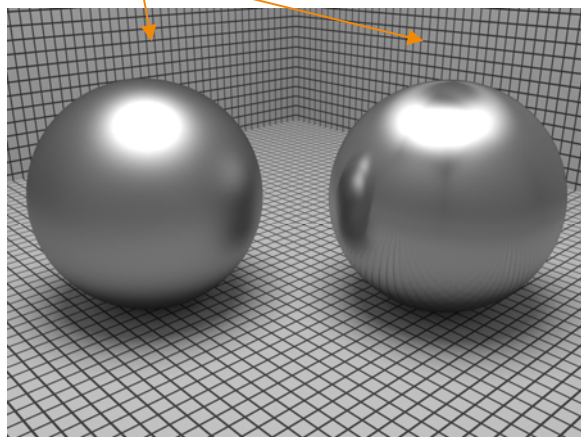
  → **develop a specific illumination model**

Types of BRDF

- **Diffuse**
  - **Reflect light uniformly over the entire reflecting hemisphere**
  - **Describe by a constant**
- **Specular**
  - **Perfect mirror**
  - **Describe by dirac delta functions**
- **Glossy**
  - **Most surfaces are neither ideally diffuse nor ideally specular**

Isotropic and Anisotropic BRDFs

- **Isotropic BRDFs depend only on**
  - θ **and**
  - $|\phi_{in} - \phi_{out}|$
  - **The appearance doesn't change as we rotate along the surface normal**
- **Anisotropic BRDFs**
  - **Brushed metal**
  - **Hair and fur**
  - **Cloth**



Empirical vs Physically-based model

- **Empirical models：**
  - **Computational efficiency for fast rendering**
  - **Approximate illumination**
  - **Special effects in movies、video art**
- **Physically-based model：**
  - **For much realistically image synthesis**
  - **Theoretical model with exact value of light energy**
  - **Lighting industry、interior decoration**

Famous BRDFs

- **Lambert's model**
  - **Only for diffuse surface**
- **Phong model / Phong-Blinn model**
  - **Not energy conservative**
- **Cook-Torrance**
  - **Modeled by geometry optics**
- **He's**
  - **Most comprehensive, expensive model**
  - **Take polarization into consideration**

Lambert's Model

- **Lambert's model**
  - **A Lambert's model describe a perfectly diffuse surface**

$$f_\lambda(\cdot) = \frac{\rho_d}{\pi}$$

Reflectance

Phong Model

- **Phong model**

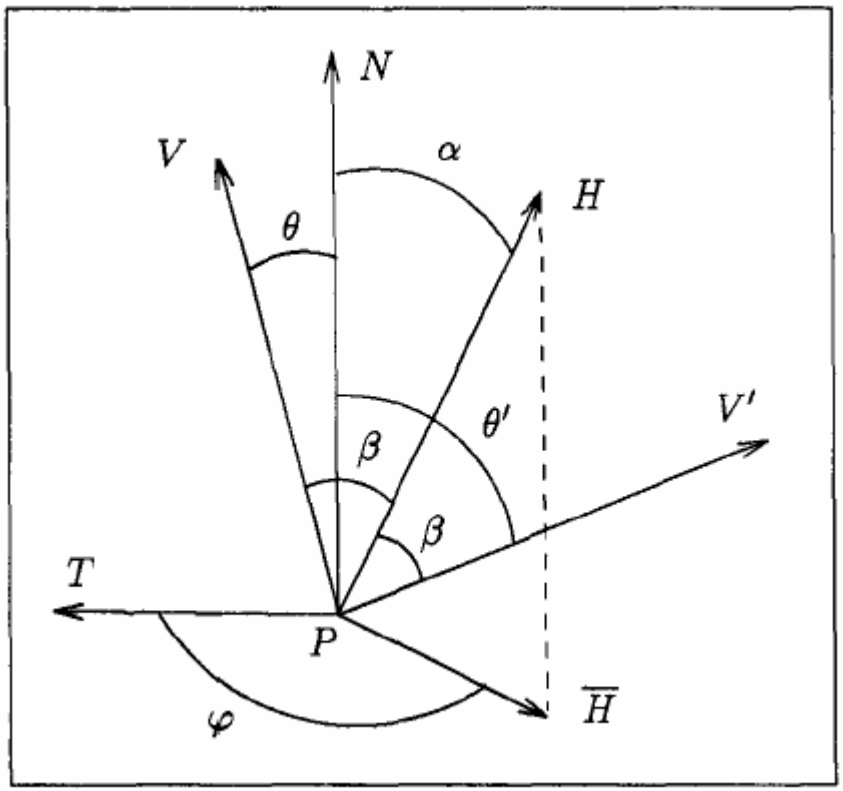
$$L(P,V) = k_a L_a + k_d \sum_{i=1}^{n} L_d^i \left(N \cdot V_i'\right) + k_s \sum_{i=1}^{n} L_s^i \left(V \cdot R_i\right)^{ns}$$

- **Blinn-Phong model**

$$L(P,V) = k_a L_a + k_d \sum_{i=1}^{n} L_d^i \left(N \cdot V_i'\right) + k_s \sum_{i=1}^{n} L_s^i \left(V \cdot H_i\right)^{ns}$$

- **Not energy conservative**

Cook-Torrance Model



V	= Outgoing direction of light
V'	= Incoming direction of light
N	= Surface normal vector
T	= Surface tangent vector*
H	= Bisector vector of V and V'
$\overline{H}$	= Projection of H i N

t	$(H \cdot N)$	α	$= \cos^{-1} t$
u	$= (H \cdot V)$	β	$= \cos^{-1} u$
v	$= (V \cdot N)$	θ	$= \cos^{-1} v$
v'	$= (V' \cdot N)$	θ'	$= \cos^{-1} v'$
w	$= (\mathrm{T} \cdot \overline{H})$	ϕ	$= \cos^{-1} w$

Cook-Torrance Model

- **With BRDF defined as**

$$f_r = sf_s + df_d$$

s+d <= 1 represent the specular and diffuse coefficient

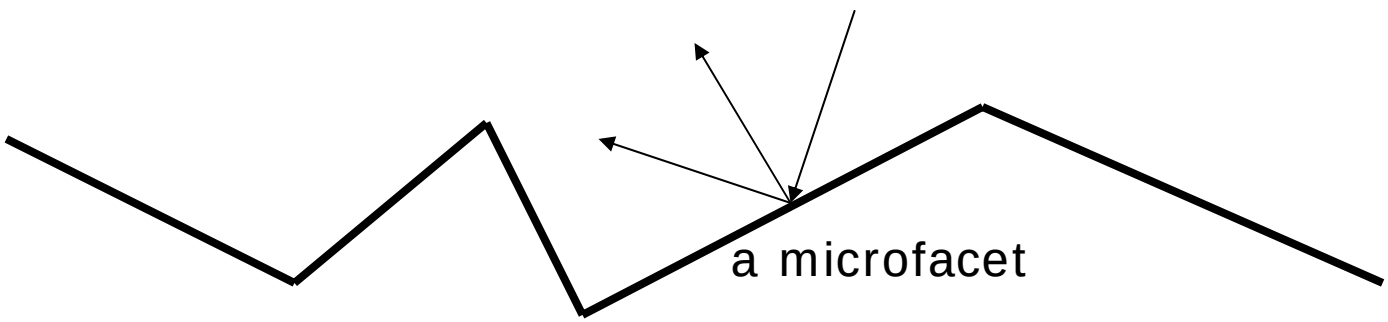
$$f_d = \frac{1}{\pi} \qquad f_s = \frac{1}{\pi} \frac{F \cdot D \cdot G}{(N \cdot V')(N \cdot V)}$$

- **Then what is F, D, G anyway?**
  - **F, D, G <= 1**

Cook-Torrance Model

- **Microfacet**
  - **microfacet：**

    a very small planar surface
  - **a rough surface is composed of many microfacets as grooves**



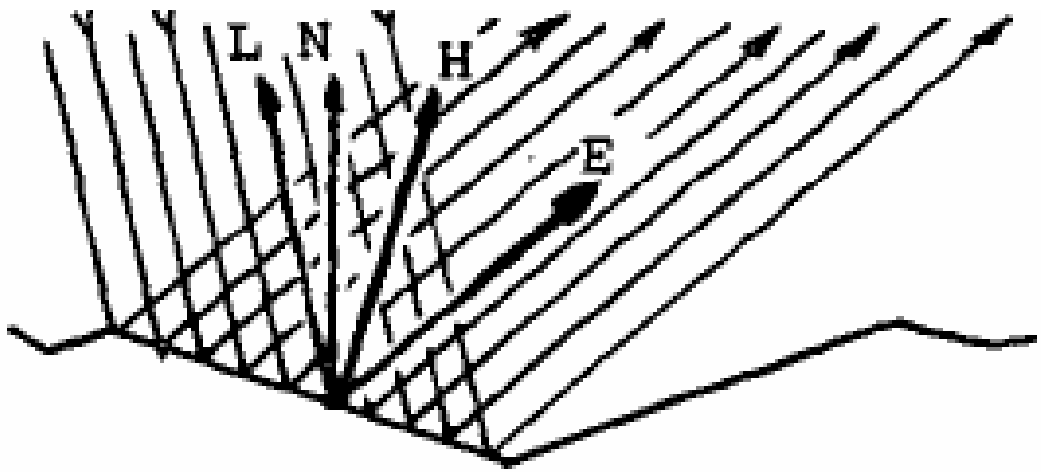
G Term

- G is the geometric attenuation coefficient which express the ratio of light that is not self-obstructed by the surface
- 3 cases
  - No interference
  - Masking
  - Shadowing

G: Geometric Attenuation -1

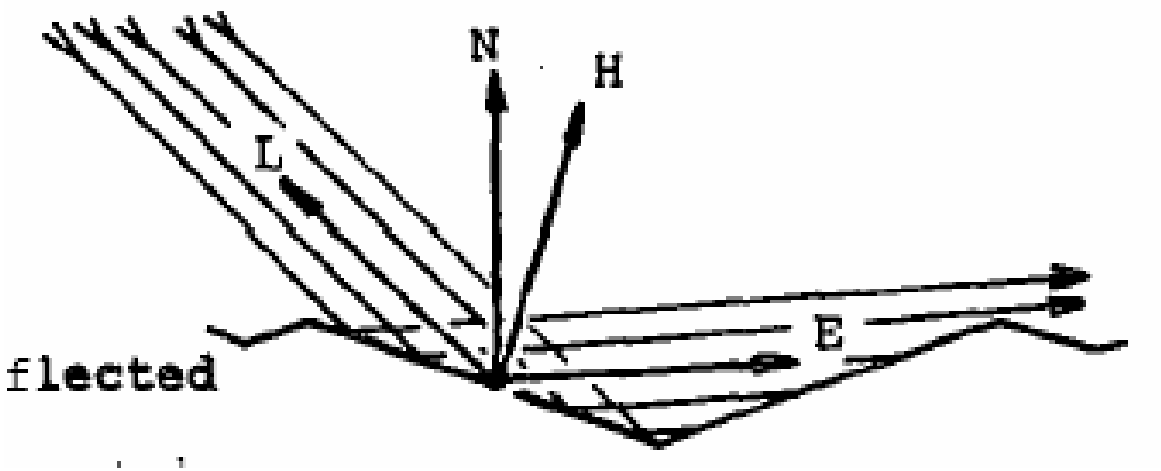
- **3 cases**

Ga
No interference

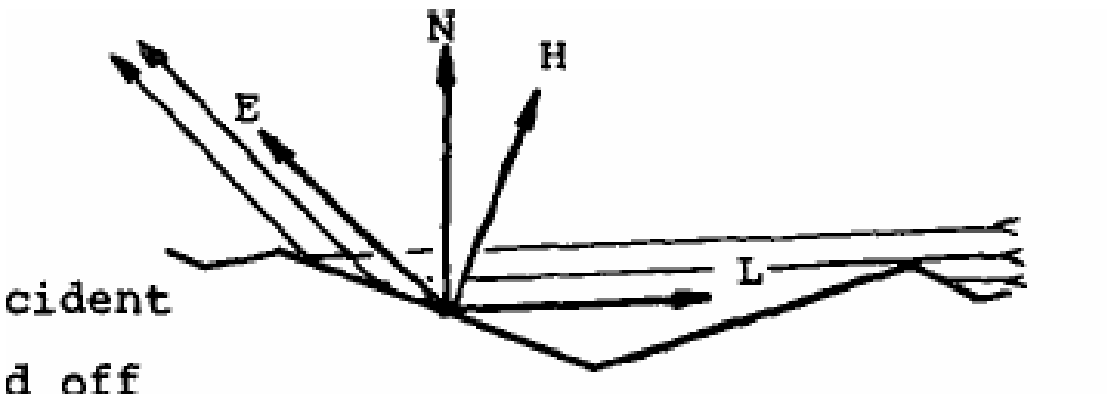


G: Geometric Attenuation -2

Gb
masking



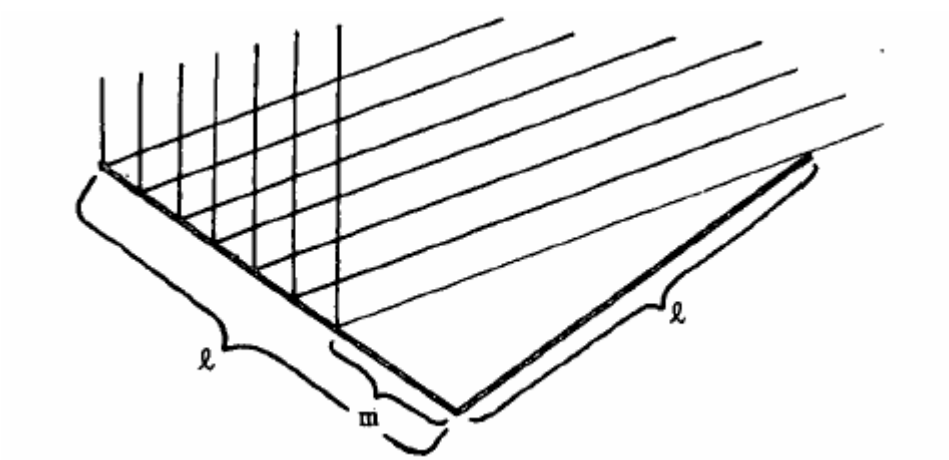
Gc
shadowing



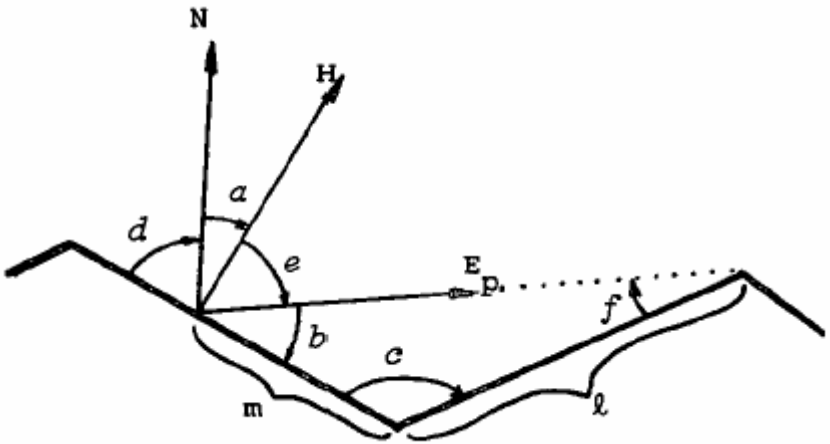
G: Geometric Attenuation -3

$$G = \min\{Ga = 1,\ Gb,\ Gc\}$$

$$Gb = \frac{\ell - m}{\ell} = 1 - \frac{m}{\ell}$$



G: Geometric Attenuation -4



$m/\ell = \sin f / \sin b$

$$\sin b = \cos e$$
$$\cos b = \sin e$$

Since the angles of the triangle must sum to 2π we have

$$\begin{aligned}\sin f &= \sin(b+c) \\ &= \sin b \cos c + \cos b \sin c\end{aligned}$$

Due to the symmetry of the groove and the complementarity of d and a

$$\begin{aligned}c &= 2d \\ \cos c &= 1-2\sin^2 d = 1-2\cos^2 a \\ \sin c &= 2\cos d \sin d = 2\sin a \cos a\end{aligned}$$

Plugging these into the expression for sinf

$$\begin{aligned}\sin f &= \cos e(1-2\cos^2 a) + 2\sin e \cos a \sin a \\ &= \cos e - 2\cos a(\cos e \cos a - \sin e \sin a) \\ &= \cos e - 2\cos a \cos(e+a) \\ &= (H\cdot E_p) - 2(N\cdot H)(N\cdot E_p)\end{aligned}$$

G: Geometric Attenuation -5

$$Gb = \frac{2(N \cdot H)(N \cdot E)}{H \cdot E}$$

→ Cos(e)

$$Gc = \frac{2(N \cdot H)(N \cdot L)}{H \cdot L}$$

$$G = \min\left\{1, \frac{2(N \cdot H)(N \cdot E)}{H \cdot E}, \frac{2(N \cdot H)(N \cdot L)}{H \cdot L}\right\}$$

D Term

- **D is the micro-facet slope distribution which defines the fraction of the facets that are oriented in direction H**
- **This describes the roughness of surface**
  - **When surface is very rough, all microfacets' orientation distributed evenly**
  - **When surface is smooth, microfacets' normal come closer to the surface normal N**

D: Slope Distribution Function

- **A Gaussian model**

$$D = ce^{-(\alpha/m)^2}$$

c: constant

$\alpha : \cos^{-1}(N \cdot H)$

m: RMS slope of the micro-facets, measured by experiments

- **A more general Beckmann distribution function**

$$D = \frac{e^{-((\tan\alpha)/m)^2}}{m^2 \cos^4 \alpha}$$

F Term

- **F is the Fresnel factor which describes how light is reflected by each smooth microfacet**
- **Fresnel term: the amount of light that is reflected as opposed to being absorbed / transmitted**
- **Fresnel Equation:**
  - **Given the index of reflection of both reflection sides**
  - **Compute the reflected amount of energy**
  - **Approximations:**
    - $F = F_0 + (1 - F_0)(1 - v.h)^5$
    - $F = (1 - v.h)^4$
    - $F = 1 / (1 + v.h)^7$